**Purpose:** In this problem set, you will improve your understanding of logarithmic functions by studying their algebraic properties.

1. Match the functions with their graphs.







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- 2. Suppose  $f(x) = 5 + \log_3(x-2)$  and  $g(x) = \log_3(x)$ .
  - (a) How can we transform the graph of g(x) into the graph of f(x)?

(b) What is the domain and range of g(x)? Does the transformation change the domain and range of f(x)? If so, list the new domain and range.

3. What is the domain of  $g(x) = \log_7(x^2 - 16)$ ?

4. Using the properties of logs, decide whether these statements are true for any A, B > 0, and write T/F next to them. If a statement is true, cite the rule. If a statement is false, give a counterexample (you may use a calculator for this part).

(a) 
$$\log\left(\sqrt{A}\right) = \frac{1}{2}\log(A)$$

(b)  $\ln(A)\ln(B) = \ln(A) + \ln(B)$ 

(c) 
$$\log(AB) = \log(A)\log(B)$$

(d) 
$$p \cdot \ln(A) = \ln(A^p)$$

(e) 
$$\frac{\log(A)}{\log(B)} = \log(A) - \log(B)$$

(f) 
$$\sqrt{\ln(A)} = \ln(A^{1/2})$$

5. WITHOUT A CALCULATOR, find the exact value of each expression below.

(a) 
$$\log\left(\frac{15}{2}\right) + \log\left(\frac{200000}{15}\right)$$

(b)  $\log_5(10) + \log_5(40) - 4\log_5(2)$ 

6. Match each of the expressions on the left with the equivalent expression on the right.

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 $100^{\log(5/2)}$	(a)	$\log\left(\frac{10}{10^2}\right)$
 $100^{\log(5)/2}$	(b)	$10^{\log(2)/2}$
 1	(c)	6.25
 $\sqrt{2}$	(d)	5
 $-\frac{1}{2}\log(2)$	(e)	$\log\left(\sqrt{2}\right)$
 $-\frac{5}{2}\log(10000)$	(f)	15
 $2 \log(100^{5/2})$	(g)	10
 $5\log(1000)$	(h)	$\log(10^{-2})$
 $\frac{1}{2}$	(i)	$10^{-\log(2)}$
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7. Suppose that  $u = \log(3)$  and  $v = \log(5)$ . Find possible formulas for the following expressions in terms of u and/or v and/or constants (that are not expressed with logs). Your answers should have no logs.

(a) 
$$\log(0.6)$$
 (c)  $\log(90)$ 

(b) 
$$\log(0.025)$$
 (d)  $\log(\sqrt{1000})$ 

8. Solve each of the equations below for x.

(a) 
$$5^{2x+1} = 7$$

(b) 
$$e^{2x} + 6 = 10$$

(c) 
$$10 + 2\ln(x - 3) = 20$$

(d) 
$$\ln(x-10) + \ln(x-2) = 0$$

(e) 
$$\log_2(x+4) + \log_2(x-3) = 3$$

(f) 
$$\ln(x-2) + \ln(2x+3) = 2\ln(x)$$

(g) 
$$\log_9(x-3) + \log_9(x+1) = \log_9(x+7)$$

(h) 
$$\log_2(x) + \log_2(x+2) - \log_2(4-x) = 2$$

(i) 
$$e^{2x} - e^x - 30 = 0$$

(j) 
$$3e^{2x} - 2e^x - 16 = 0$$