$\qquad$
Precalculus
Purpose: In this problem set, you will improve your understanding of logarithmic functions by studying their algebraic properties.

1. Match the functions with their graphs.


A


C


B


D
$\qquad$

$$
f(x)=\log _{1 / 2}(x)
$$

$\qquad$

$$
f(x)=2^{x}
$$

$$
f(x)=\log _{2}(x)
$$

$$
\ldots f(x)=\left(\frac{1}{2}\right)^{x}
$$

2. Suppose $f(x)=5+\log _{3}(x-2)$ and $g(x)=\log _{3}(x)$.
(a) How can we transform the graph of $g(x)$ into the graph of $f(x)$ ?
(b) What is the domain and range of $g(x)$ ? Does the transformation change the domain and range of $f(x)$ ? If so, list the new domain and range.
3. What is the domain of $g(x)=\log _{7}\left(x^{2}-16\right)$ ?
4. Using the properties of logs, decide whether these statements are true for any $A, B>0$, and write $T / F$ next to them. If a statement is true, cite the rule. If a statement is false, give a counterexample (you may use a calculator for this part).
(a) $\log (\sqrt{A})=\frac{1}{2} \log (A)$
(b) $\ln (A) \ln (B)=\ln (A)+\ln (B)$
(c) $\log (A B)=\log (A) \log (B)$
(d) $p \cdot \ln (A)=\ln \left(A^{p}\right)$
(e) $\frac{\log (A)}{\log (B)}=\log (A)-\log (B)$
(f) $\sqrt{\ln (A)}=\ln \left(A^{1 / 2}\right)$
5. WITHOUT A CALCULATOR, find the exact value of each expression below.
(a) $\log \left(\frac{15}{2}\right)+\log \left(\frac{200000}{15}\right)$
(b) $\log _{5}(10)+\log _{5}(40)-4 \log _{5}(2)$
6. Match each of the expressions on the left with the equivalent expression on the right.
$\qquad$ $-2$
$\ldots 100^{\log (5 / 2)}$
(a) $\log \left(\frac{10}{10^{2}}\right)$
_ $100^{\log (5) / 2}$
(b) $10^{\log (2) / 2}$
$-\quad-1$
(c) 6.25
$\longrightarrow \sqrt{2}$
(d) 5
$=\frac{1}{2} \log (2)$
(e) $\log (\sqrt{2})$
$=\frac{5}{2} \log (10000)$
(f) 15
$\qquad$ $\log \left(100^{5 / 2}\right)$
(g) 10
$\qquad$ $5 \log (1000)$
(h) $\log \left(10^{-2}\right)$
$\longrightarrow \frac{1}{2}$
(i) $10^{-\log (2)}$
7. Suppose that $u=\log (3)$ and $v=\log (5)$. Find possible formulas for the following expressions in terms of $u$ and/or $v$ and/or constants (that are not expressed with logs). Your answers should have no logs.
(a) $\log (0.6)$
(c) $\log (90)$
(b) $\log (0.025)$
(d) $\log (\sqrt{1000})$
8. Solve each of the equations below for $x$.
(a) $5^{2 x+1}=7$
(b) $e^{2 x}+6=10$
(c) $10+2 \ln (x-3)=20$
(d) $\ln (x-10)+\ln (x-2)=0$
(e) $\log _{2}(x+4)+\log _{2}(x-3)=3$
(f) $\ln (x-2)+\ln (2 x+3)=2 \ln (x)$
(g) $\log _{9}(x-3)+\log _{9}(x+1)=\log _{9}(x+7)$
(h) $\log _{2}(x)+\log _{2}(x+2)-\log _{2}(4-x)=2$
(i) $e^{2 x}-e^{x}-30=0$
(j) $3 e^{2 x}-2 e^{x}-16=0$
